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PRICING, DEFAULT RISK AND OPTIMAL USE

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# Forward Contracts: Pricing, Default Risk and Optimal Use

Ian A. Cooper\* and Antonio S. Mello\*\*

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## Abstract:

This paper addresses two related issues: the equilibrium pricing of default risk in foreign exchange forward contracts and the optimal use of such contracts for hedging given that the forward price reflects default risk. A model is developed where the motivation for hedging is to avoid deadweight losses associated with bankruptcy. In this model, not all firms that would hedge if forward contracts were default-free will be willing to hedge. We show which firms should hedge and discuss the relationship between forward hedging and hedging with futures.

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Forward exchange markets are used extensively for hedging foreign exchange risk, (Levi and Zechner (1989)). The price quoted for forward exchange transactions includes a bid-ask spread which varies with the maturity of the contract. This spread is, at least partly, to compensate the bank offering the forward contract for default risk. Thus to fully explain forward exchange pricing it is necessary to have a model of the pricing of forward contract default risk.

Models of optimal hedging using forward exchange contracts such as Smith and Stulz (1987) typically assume that the forward contract is either priced ignoring default risk, or that the price is set exogenously to the particular hedging application for which it is used. Neither assumption is particularly appealing. The former simply ignores the issue; the latter raises complex questions of moral hazard and potential rationing since low grade (high default risk) hedgers will, presumably, find a particular rate more appealing than high grade hedgers.

The hedging behaviour of forward contracts can also be replicated by a variety of other transactions. These include foreign exchange futures contracts, international borrowing/lending transactions, and currency swaps. These transactions perform at least two functions related to forward contracts. They provide the banks offering forward contracts the means to hedge the resulting rate exposure. They also provide substitution possibilities for the users of forward contracts.

A complete model of foreign exchange forward pricing and use must, therefore:

1. Explain the equilibrium pricing of default risk.
2. Derive optimal hedging strategies when forward prices are set to include default risk.
3. Explain why forward contracts will be preferred to other hedging instruments by some hedgers.

The purpose of this paper is to examine these three issues.

The paper is organised as follows. Section 2 states the basic assumptions used throughout the paper. Section 3 derives distribution-free results. Section 4 examines the default risk spread in the forward market, and its relationship to the spread in the debt market. Section 5 gives a simple analytically tractable model to illustrate which firms will hedge. Section 6 examines the impact of hedging with more realistic assumptions. Section 7 contains the summary and conclusions.



## Section 2: ASSUMPTIONS

We are concerned with a corporation that is attempting to hedge the economic exposure arising from the fact that part of its value is correlated with the value of a foreign currency. This could arise in many ways, including the ownership of foreign assets, foreign currency sales or competitors based in the foreign currency area. The source of the risk is not important, but the reason for hedging is important to us. We assume that the corporation is hedging to raise its equity value by avoiding bankruptcy costs. This is the simplest to model of the three reasons given by Smith and Stulz (1985) for corporate hedging.<sup>1</sup>

The corporation with which we are concerned is similar to the corporation analysed by Merton (1974). It has a collection of assets with current value  $V_0$  and zero coupon debt of face value  $D$  maturing at a future date  $T$ . The corporation will pay no dividends to its equity holders prior to  $T$ . If, at  $T$ , the value of the assets,  $V_T$ , is below the promised debt payment,  $D$ , then there is a deadweight loss of  $K$  as the corporation goes through bankruptcy and the residual assets are claimed by the debtholders.<sup>2</sup>

The assumptions about the corporation prior to hedging are, therefore:

- A1: The corporation has assets with current value  $V_0$  and random value at time  $T$  of  $V_T$ .
- A2: The corporation has a single zero coupon debt claim payable at  $T$  with face value  $D$ .
- A3: The corporation will pay no dividends and raise no new equity prior to  $T$ .
- A4: In the event of default, there will be deadweight loss of  $K$ , and the remaining value will be claimed by the debtholders.

With these assumptions, the payoffs to the debt and equity holders are given in Table 1, and the values of the equity and debt are:

$$E_0 = C(V_T, D) \tag{1}$$

$$B_0 = D/(1+R_D) - P(V_T, D) - KW(V_T < D) \tag{2}$$

where:

$E_0$  is the value of the equity

$B_0$  is the value of the debt

$R_D$  is the riskless total interest rate between time zero and time  $T$

$C(V_T, D)$  is the value of a European call option maturing at  $T$  with an exercise price of  $D$ , exercisable into the asset  $V$ .

$P(V_T, D)$  is the value of a European put option maturing at  $T$  with an exercise price of  $D$ , exercisable in the asset  $V$ .

$W(V_T < D)$  is the value of a claim that pays at  $T$  conditional on  $V_T < D$ .

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Table 1 approximately here  
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The motive for hedging in this case will be to reduce the expected cost of bankruptcy. The hedging vehicle is a forward exchange contract on a face value of foreign currency of an amount  $F$ . The contract exchange rate is  $X_F$  (dollars per unit of foreign currency). At the maturity date of the contract, time  $T$ , if no default occurs the corporation will receive a net amount equal to  $F(X_T - X_F)$  dollars, where  $X_T$  is the random spot rate for this foreign currency at time  $T$ . This is summarised by the following assumptions:

A5: The corporation can costlessly enter a forward exchange agreement to pay  $FX_F$  and receive  $FX_T$  at time  $T$ .

A6: The supply of forward contracts is competitive

We will later make assumptions that ensure that  $V_T$  is correlated negatively with  $X_T$ , so that this transaction is a hedge which potentially reduces the probability of bankruptcy. Finally, we assume perfect markets, no taxes and continuous price paths:

A7: Capital markets are perfect and competitive. Trading is continuous and costless. There are no taxes. Asset values and exchange rates follow diffusion processes.

This completes the basic assumptions of the model. We now proceed to analyse the effect of default risk on the forward contract price and the optimal amount of hedging.

### Section 3: PAYOFFS TO CLAIM HOLDERS AND HEDGING

There are two aspects of the forward contract settlement that will be important in modelling default risk. One is whether the contract is treated as an exchange of gross

amounts or net amounts. The other is the priority structure of claims in default. Throughout, we make the assumption:

A8: The bank granting the forward contract is riskless

We analyse three alternative settlement rules:

Assumption AG: Gross Settlement.

Settlement of the forward contract will occur by the "bank" paying the corporation the amount  $FX_T$  and then becoming a creditor for the amount  $FX_F$ . The bank's claim is of equal priority to the debt claim.

Assumption ANE:

Settlement of the forward contract will occur by the "bank" paying the corporation the amount  $(FX_T - FX_F)$  if it is positive. If  $(FX_T - FX_F)$  is negative, the bank becomes a creditor for this amount with priority equal to the debt holders.

Assumption ANJ: Net settlement, junior priority.

As assumption ANE except that the bank's claim is junior to the debt claim.

We now proceed to derive the distribution-free payoffs to the various claimants on the company and then solve for equilibrium forward pricing.<sup>3</sup>

Under the assumption AG, the sequence of events is shown in Table 2A. The payoffs to the three claimants: debt, equity and the bank holding the forward contract are shown in Table 2B. The debt claim has a payoff of  $D$  unless default occurs, in which case it receives its share of the value of the firm's assets plus the payment of  $FX_T$  minus the deadweight loss,  $K$ . The forward claim always loses  $FX_T$  and then receives back  $FX_F$  if there are sufficient assets. If not, the forward claim receives back its share of the firm's assets net of default costs. The equity simply receives any surplus once the debt and forward claims have been settled. The aggregate of all claims is the value of the firm's assets less any deadweight loss arising from bankruptcy.

The value of the claims are given by:

$$B_1 = dD - [D/(D+FX_F)](P(V_T+FX_T, D+FX_F) + KW(V_T+FX_T < D+FX_F)) \quad (3)$$

$$H = -FX_0 + dFX_F [FX_F/(D+FX_F)](P(V_T+FX_T, D+FX_F) + KW(V_T+FX_T < D+FX_F)) \quad (4)$$

$$E_1 = C(V_T+FX_T, D+FX_F) \quad (5)$$

where:

$$d = 1/(1+R_D)$$

(6)

$B_1$  is the value of the debt, given a forward contract of face value  $F$

$H$  is the value of the forward contract

$E_1$  is the value of the equity given a forward contract of face value  $F$

$X_0$  is the value of a default free claim that pays  $X_T$  at time  $T$

In addition, it is straightforward to see that:

$$B_1 + H + E_1 = V_0 - KW(V_T + FX_T < D + FX_F) \quad (7)$$

Thus the total value of all claims is equal to the value of the assets of the firm minus the present value of expected bankruptcy costs.

Tables 3 and 4 show the payoffs to the claim holders under the alternative settlement rules ANE and ANJ. Both of these rules involve net settlement, so that the bank pays  $(FX_T - FX_F)$  if it is positive, and is a creditor for  $(FX_F - FX_T)$  if  $FX_T$  is less than  $FX_F$ . The rules differ from each other in that the bank's claim is subordinated to the debt claim under rule ANJ, whereas it has equal priority under rule ANE.

The issue in hedging is whether the equity value,  $E_1$ , can be raised by initiating a forward contract. It will turn out that the constraints imposed by the debt holders are crucial in determining whether hedging is viable. We use two alternative assumptions about the behaviour of debt holders:

- A9: Debt holders price their debt assuming that no hedging will occur, but write covenants to protect their claim against any fall in value as a result of "hedging"
- A9': Debt holders price their debt claim including the effect of the optimal hedging policy, and can costlessly bond and monitor the managers pursuing this policy.

The difference between these two assumptions is that the former excludes any hedging behaviour that expropriates the debt holders' claim, but does allow hedging to affect the wealth of debt holders. Thus the only impact of hedging on the wealth of the debt holders that is possible under A9 is that debt holders are subsidised by hedging.

We now define a viable hedging strategy under A9:

Definition:

Under A9, a viable hedging strategy is a pair,  $(F, X_F)$ , such that:



1.  $H = 0$  (equilibrium in the forward market)
2.  $E_1 > E_0$  (hedging increases the equity value)
3.  $B_1 \geq B_0$  (hedging does not expropriate debt holders)

Under A9', a viable hedging strategy is one which reduces the expected loss from bankruptcy. If debt holders price the debt to include the optimal hedging strategy, raising the value of the equity will be equivalent to raising the total value of the firm, which is equal, given the equilibrium condition that  $H=0$ , to:

$$E_1 + B_1 = V_0 - KW(V_T + FX_T - D + FX_F) \quad (8)$$

Thus, under A9, a viable hedging strategy is a pair,  $(F, X_F)$ , such that:

1.  $H = 0$
2.  $E_1 + B_1 > E_0 + B_0$

The sequence of analysis will be as follows. We shall first impose the competitive condition,  $H=0$ , to determine the equilibrium forward contract price,  $X_F$ , corresponding to a particular amount of hedging,  $F$ . We shall examine the behaviour of  $X_F$  as  $F$  varies. We shall then derive the impact on the equity value of hedging with different levels of  $F$ . Finally, we shall derive the amount of hedging that maximises the equity value,  $E$ . Throughout, the values  $E_1$  and  $B_1$  and the contract rate,  $X_F$ , will be functions of the amount of hedging,  $F$ . The impact of hedging will be to:

- a) Reduce the probability of bankruptcy and the associated expected deadweight loss.
- b) Change the risk of the debt and its value.

Thus the total value  $(B_1 + H + E_1)$  will rise as the corporation hedges. The equilibrium condition,  $H=0$ , means that  $(B_1 + E_1)$  will rise. Under A9,  $B_1$  will itself possibly rise if the debt becomes less risky, so that the net effect on the equity value will depend on whether the gain from avoiding bankruptcy is greater or less than the wealth transfer to debt holders.

This leads to the following proposition:

**Proposition P1:** For viable hedging, the risk-adjusted probability of bankruptcy must fall.<sup>4</sup>

**Proof:** Under A9 and A9' viable hedging requires:

$$B_1 + E_1 > B_0 + E_0 \quad (9)$$

Which implies:

$$W(V_T + FX_T < D + FX_F) < W(V_T < D) \quad (10)$$

Both claims  $W$  are spanned by the assets  $V$  and  $X$ , so that their values are proportional to the risk-adjusted probabilities of default.

This completes the proof.

Proposition P1 states that hedging will be viable only if it decreases the probability of bankruptcy. The condition that debt is not expropriated is important in this regard. If we did not impose this condition, "hedging" could increase equity values by simply taking the form of speculation that increases the variance of  $(V_T + FX_T)$  and hence the value of the equity which is a call option on this variable. In this case, however, the "hedging" would be simply a way of expropriating the wealth of debt holders. Note that this sufficient condition for viable hedging is distribution-free. It applies to any distributional forms of  $V_T$  and  $X_T$  that have continuous price paths.

Under A9 the following proposition obviously follows:

**Proposition P2:** With costless bonding and monitoring by the debt holders (A9), the optimal hedging strategy is the one that minimises the risk-adjusted probability of bankruptcy.

**Proof:** Obvious from the maximisation of (8)

In some cases, the bank granting the forward contract may own some of the existing debt of the firm. Under A9, such banks will be willing to grant the forward contract at a better price than banks not holding debt. The negative NPV of the forward contract will be offset by a gain from the subsidy to the debt holding.

We define the minimum rate charged by a bank holding a proportion  $a$  of the firm's debt by  $X_F(a)$ , the solution to:

$$a(B_1 - B_0) + H = 0 \quad (11)$$

Where  $B_1$  and  $H$  are evaluated at  $FX_F(a)$ . We now prove:

**Proposition P3:** The forward rate offered,  $X_F(a)$ , is non-increasing in  $a$ .

**Proof:**  $H$  is increasing in  $X_F$ ;  $B_1 > B_0$ ;  $H=0$  at  $X_F$ ; so (11) is satisfied at  $X_F(a) < X_F$ .

Corollary: The best forward rate will be offered by the bank holding the most debt.

Proposition P3 and its corollary demonstrate a common feature of forward markets.

Forward contracts are typically granted by banks that already have a relationship with the hedger. In this model that is because the forward market is an imperfect instrument. It hedges the firm, in the sense of decreasing the probability of default but, as a direct consequence of this, it also subsidises the existing debt holdings.

Note that the phenomenon here is what is commonly known as "relationship banking", whereby certain contracts are offered more competitively by those banks having a pre-existing relationship with the firm, in the sense that they already own claims on the firm. The motivation for relationship banking in this case is that the granting of the forward contract inevitably increases the value of the existing debt claim. One can imagine many situations of this type where the best rate will be offered on a new contract by a bank holding a security that will be subsidised by the granting of the new contract.

It is, perhaps, worth pointing out what type of contract would not have this effect. Such a contract would have to reduce the probability of default without subsidising the existing debt holders. It is hard to see exactly what such a contract could look like.

#### SECTION 4: Default risk spreads in the forward market

Under AG, the relationship between the default spreads on the debt and on the forward contract is very simple. We define:

$$s_D = \ln (dD/B_1)/T \quad (12)$$

$$s_F = \ln (X_F/M_X)/T \quad (13)$$

where  $M_X$  is the equilibrium default-free forward rate.  $s_D$  is the continuous annual rate spread for default risk on the debt after the forward contract is undertaken, and  $s_F$  is the equivalent for the forward contract.<sup>5</sup> We now demonstrate:

**Proposition P4:** Under AG, the equilibrium forward rate, including the possibility of default, is set by adding the equivalent of the debt market default spread to the riskless forward quote:

$$s_F = s_D \quad (14)$$

**Proof:**

$$H = -FX_0 + dFX_F - (FX_F/D)(B_1 - dD) \quad (15)$$

In Equilibrium:

$$H = 0 \text{ and } X_0 = dM_X \quad (16),(17)$$

So:

$$(dFX_F - FX_o)/FX_F = (B_1 - dD)/D \quad (18)$$

$$(X_F - M_X)/X_F = (B_1 - dD)/dD \quad (19)$$

$$X_F/M_X = dD/B_1 \quad (20)$$

This completes the proof.

Note that the forward spread is equal to the debt spread after the forward contract is undertaken. If the forward contract subsidises the debt, the forward spread will be less than the debt spread prior to hedging. This result is not surprising, as the payoffs to the bank holding the forward contract are identical to those resulting from a transaction where it borrows debt of a face value  $FX_T$  and lends debt of face value  $FX_F$  to the hedging firm.

With net settlement (rules ANE and ANJ) spreads in the forward market will, in general, be different to spreads in the debt market. In the case settlement rule ANE, the direction of the difference is unambiguous:

**Proposition P5:** Under ANE, the forward spread will be less than the debt market spread after hedging.

**Proof:** Comparison of the payoffs to the forward contract in Tables 2 and 3 shows that the payoff to the bank under ANE is always at least as great as that under AG, and is greater in some states.  $H$  is increasing in  $X_F$ , so the forward rate under ANE is less than that under AG.

This completes the proof.

This proposition results from the fact that net settlement (ANE) is more favourable to the bank than gross settlement, as the exposure to default is limited to the net difference between  $FX_T$  and  $FX_F$ , rather than the gross amount  $FX_F$ .

Under ANJ there are two differences to AG. One is net settlement which will make the spread lower. The second is the juniority of the forward claim, which will make the spread higher. Neither of these effects will, in general, dominate, so we cannot say whether the forward spread under ANJ will be greater or less than the debt spread.

## Section 5: NORMALLY DISTRIBUTED PAYOFFS

The most commonly used assumptions for the distributed form of  $V_T$  and  $X_T$  are that they are lognormal. Analysis with this assumption is possible only numerically, and is given in Section 6. To illustrate the general nature of the results, we first make the more tractable assumption that  $X_T$  and  $V_T$  are jointly normal (ie.  $X_t$  and  $V_t$  perform

arithmetic Brownian motion).

$$A10: V_T \sim N(M_V, S_V) \quad (21)$$

$$A11: X_T \sim N(M_X, S_X) \quad (22)$$

$$A12: \text{Corr}(X_T, V_T) = r \quad (23)$$

Combined with the continuous trading assumption, this assumption allows us to price any claim contingent on only  $V$  and  $X$  as if the world is risk-neutral (Cox, Ross and Rubinstein(1979)).

Thus we can set:

$$dM_V = V_o \quad (24)$$

$$dM_X = X_o \quad (25)$$

We value all claims by taking expected payoffs and multiplying by  $d$ . Solving for  $H$  in this way and imposing the equilibrium condition,  $H=0$ , gives (under AG):

$$-FM_X + FX_F - [FX_F/(D+FX_F)][(D+FX_F+K \cdot M_V - FM_X)N_1 + S_Z N_1'] \quad (26)$$

Where:

$$N_1 = N[(D+FX_F - M_V - FM_X)/S_Z] \quad (27)$$

$$N_1' = N'[(D+FX_F - M_V - FM_X)/S_Z] \quad (28)$$

$N(.)$  is the standard normal distribution function

$N'(.)$  is the standard normal density function

$$Z_T = V_T + FX_T \quad (29)$$

$$M_Z = M_V + FM_X \text{ is the mean of } Z_T \quad (30)$$

$$S_Z = (S_V^2 + F^2 S_X^2 + 2FS_V S_X)^{1/2} \text{ is the standard deviation of } Z_T \quad (31)$$

Evaluation of  $E_1$  and  $B_1$  gives:

$$E_1/d = (M_V + FM_X - D - FX_F)(1 - N_1) + S_Z N_1 \quad (32)$$

$$B_1/d = D - [D/(D+FX_F)][(D+FX_F+K \cdot M_V - FM_X)N_1 + S_Z N_1] \quad (33)$$

Summing the values gives:

$$B_1 + H + E_1 = V_0 - dKN_1 \quad (34)$$

Expressions (32) and (34) hold regardless of the settlement rule, and the following two propositions hold for all rules.

Proposition P6 demonstrates, under the normality assumption, that no firms will find hedging viable.

**Proposition P6:** There are no viable hedges without bonding if the bank granting the forward contract owns no debt of the hedging firm.

**Proof:** Appendix

Proposition P6 demonstrates, under relatively weak assumptions, that hedging will not be viable using forward contracts if the bank granting the forward contract does not own any of the debt of the hedging company and the hedging company cannot bond itself to hedge. The reason, given our assumptions, is fairly straightforward. As demonstrated by Proposition P1, hedging must reduce the probability of bankruptcy to be viable, in the sense that it does not expropriate debt holders. If hedging reduces the probability of bankruptcy it will subsidise debt holders and increase the combined value of debt and equity. It is not possible, however, for the increase in the total value of the firm to exceed the subsidy to debt holders, so viable hedging is impossible.

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 Figures 1 and 2 approximately here  
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Figures 1 and 2 show numerical illustrations of this effect. The firm involved has assets with current value of 100 and volatility of 40% per annum. It is hedging a currency with a volatility of 15% per annum and the maturity of the debt and forward contract are both five years. The deadweight loss parameter,  $K$ , is 5, and the face value of the debt is 50. Figure 1 shows the total expected value at the maturity date of the combined debt and equity claims for correlations,  $r$ , of -1, 0, 1. For a correlation of -1 between  $V$  and  $X$ , the value of the firm rises as it hedges because the total variance of the hedged firm falls as a result of hedging. The equity value of this firm, shown in Figure 2, falls, however. The reduction in the variability of the hedged asset makes the call option feature of the equity less valuable, so that the increase in the firm value benefits only the debtholders. In the case where the correlation is +1, the call option feature of the equity is made more valuable by "hedging" but the value of the equity falls because the total value of the firm is falling faster than the increase in equity value as a result of its option characteristic.

Given that the bank holding some of the existing debt of the firm will offer a better forward rate, it might seem that there is a viable hedging strategy if a bank owns enough debt. This is certainly the case, but it is still a situation which is unlikely to be seen, as a result of the following proposition:

**Proposition P7:** Even when banks hold some of the existing debt, there will be no viable hedging at a positive spread in the forward market if there is no bonding.

**Proof:** Appendix

We now turn to the conditions under which there will be viable hedging at a positive spread. These are:

1. Debt is priced at the time of issue to reflect an "optimal" hedging policy and the monitoring and enforcement of this policy is costless.
2. Equity holders or their agents, the managers, derive utility from a reduction in the variance of the firms asses, independent of an impact on the share price.

When the holders of the debt of the firm can costlessly bond the firm to pursue a particular hedging strategy and costlessly monitor this contract, they will be willing to price the debt to include the impact of the hedging strategy. Thus the problem becomes one of choosing the hedging strategy,  $F$ , to maximise the value of equity given that the debt is priced to have zero NPV including the impact of hedging. Since the forward contract, by assumption, has zero NPV, maximising the equity value is equivalent to maximising:

$$B_1 + H + E_1 = V_0 - dKN_1 \quad (35)$$

Note that now hedging may be viable at a positive spread, because the forward contract rate will be set by equation (16) rather than equation (11). In the case where the bank granting the forward contract holds some of the debt, hedging is made viable by offering a more advantageous forward rate, as reflected in (11), which leads ultimately to a negative spread for viable hedging. In this case, the forward spread is positive because of the chance of forward contract default, but hedging is made viable because the holders of debt are willing to pay a price that reflects the value of hedging.

Firms are trying to maximise (35) in this case, so they will be willing to hedge if they can find a value of  $F$  for which the probability of default with hedging,  $N_1$ , is less than the probability of default without hedging,  $N_0$ . We now generate a sufficient condition for firms to be willing to hedge, under AG:

**Proposition P8:** Under AG, a sufficient condition for some hedging is that:

$$r < -[(X_F - M_X)S_V]/[S_X(M_V - D)] \quad (36)$$

Where  $X_F$  is evaluated at  $F=0$

**Proof:**

The firm will be willing to undertake a forward contract of face value  $dF$  if:

$$dN_1/dF < 0 \text{ evaluated at } F=0$$

$N_1$  is monotonic in  $[(D+FX_F-FM_X-M_V)/S_Z]$

$$d[(D+FX_F-FM_X-M_V)/S_Z]/dF =$$

$$(X_F-M_X+FX_F)/S_Z - (FS_X+rS_VS_X)(D+FX_F-FM_X-M_V)/S_Z \quad (37)$$

Evaluating (37) at  $F=0$ , the sufficient condition for hedging becomes:

$$(X_F-M_X)/S_V - rS_VS_X(D-M_V)/S_V^3 < 0 \quad (38)$$

Rearranging gives (36).

The importance of proposition P8 is made clear from the following corollary:

Corollary:

Other things being equal, firms will be more likely to hedge if:

- a) They can find a contract with high correlation (large  $|r|$ )
- b)  $(X_F-M_X)$  is small
- c)  $S_V$  is small
- d)  $S_X$  is large
- e)  $(M_V-D)$  is large

Note that (b), (c) and (e) imply that default risk is low, (a) implies that the hedging instrument is highly correlated with the firm's assets, and (d) implies that the hedging instrument gives a lot of hedging per dollar of face value. In addition, (b) could be caused by a settlement or priority rule that reduces the forward contract default risk. Thus the firms that we would expect to see hedging in the forward markets are those with assets highly correlated to the available forward instruments, and low default risk. Other firms may wish to hedge, but the terms on which they can hedge  $(X_F-M_X)$  do not allow them to do so with positive value to their equity holders. Not all firms hedge even with bonding because the spread  $(X_F-M_X)$  can increase the probability of bankruptcy even if  $\text{Var}(Z)$  is falling.

These other low quality firms may, presumably, hedge using other instruments such as futures. For a properly margined futures contract there will be no default risk, so that (35) becomes:



$$r' < 0$$

(39)

where:

$r'$  is the correlation between the futures price and  $V$ .

Note that  $|r'|$  will, in general, be lower than  $|r|$  for two reasons. First, futures contracts have basis risk. Second, futures contracts are written on a smaller range of currencies than forward contracts. Thus, even if there is a forward contract for which  $r < 0$  there may not be a futures contracts for which  $r' < 0$ . Note also that, although (39) appears a less stringent condition than (35), it does not mean that high quality hedgers should use the futures market rather than the forward market, since we would normally expect that  $|r'| < |r|$ .

We now examine the optimal level of hedging and compare it with a commonly proposed hedging strategy, that of minimising the variance of the firm's assets including the hedge. The optimal hedge amount, if it exists, satisfies:

$$dN_1/dF = 0 \quad (40)$$

Which implies:

$$dA/dF = 0 \quad (41)$$

Where:

$$A = (D + FX_F - FM_X - M_V)/S_Z \quad (42)$$

Differentiating gives:

$$(dA/dF) = [X_F - M_X + F(dX_F/dF)]/S_Z - (A/S_Z)dS_Z/dF \quad (43)$$

The condition for minimum variance is:

$$dS_Z/dF = 0 \quad (44)$$

The equilibrium condition is:

$$(dH/dF) = [X_F - M_X + F(dX_F/dF)]$$

$$-[D(X_F + F(dX_F/dF))(FX_F - FM_X)]/[FX_F(D + FX_F)]$$

$$-FX_F[N_1(X_F - M_X + F(dX_F/dF))] = 0 \quad (45)$$

which implies:

$$\text{Sign}(X_F - M_X - F(dX_F/dF))$$

$$= \text{Sign}(1 - D(FX_F - FM_X)/[FX_F(D + FX_F)] - FX_F N_1/(D + FX_F)) \quad (46)$$

At the minimum variance value of  $F$ , from (43) and (44):

$$dA/dF = X_F - M_X - F(dX_F/dF) \quad (47)$$

Combining (46) and (47); at the minimum variance value of  $F$ :

$$\text{Sign}(dN_1/dF) = \text{Sign}((FX_F)^2(1 - N_1) + DFM_X) > 0 \quad (48)$$

Thus the minimum variance hedge ratio results in an increasing value of  $N_1$ . If there is a value of  $F$  that gives a minimum for  $N_1$ , it must be below the minimum variance value of  $F$ . This proves the following proposition:

**Proposition P9:** The optimal amount of hedging under AG is less than the minimum variance hedge ratio.

The main reason for proposition P9 is that the forward rate,  $X_F$ , is not equal to the riskless forward rate,  $M_X$ . When the firm hedges, it promises to pay a spread,  $(X_F - M_X)$ . This spread becomes a liability of the firm, and affects the bankruptcy probability. As the amount of hedging,  $F$ , rises and decreases the variance of  $Z$ , the spread is offsetting part of the effect of the falling variance on the default probability. Thus the optimal level of hedging occurs where  $dZ/dF < 0$ .

## SECTION 6: Lognormal payoffs

In section 5 we analysed the behaviour of the model with a normally distributed firm value and exchange rate. Such an assumption leads to a finite probability of negative values for these variables and is, therefore, unappealing. In this section we investigate whether the general results derived with normal distributions hold with the more realistic assumption of lognormal distributions.

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Tables 3 - 17 approximately here  
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To do so we numerically integrate the payoff functions given in Tables 2-4, using the trapezoidal quadrature given in Press, Flannery, Tenkolsky and Vetterling (1986). Tables 3-14 shows the resulting values of debt, equity, firm value and the contract rate,  $X_F$ , for representative parameter values and varying levels of hedging,  $F$ .

In all cases, for the parameter values we have investigated, the results derived with normal distributions remain true with the more realistic lognormal assumption. In particular, there is still, given the other parameter values, a minimum absolute correlation,  $H$ , that is necessary to generate some hedging. The size of the necessary correlation varies with the settlement rule and is 0.43, 0.23 and 0.16 for rules AG, ANJ and ANE respectively. Thus for all rules a significantly positive correlation between the assets of the firm and the hedging instrument is necessary to generate hedging.

In Figures 3 -6 the gross settlement rule, AG, is examined under the assumption that the firm value and the exchange rate follow lognormal processes. In Figure 3, the firm value increases as a result of hedging if the currency has a negative correlation with the asset value. Figures 4 and 5 show the effects of this upon equity and debtholders. In Figure 4, the equity value is decreasing as the firm hedges with the currency that has a negative correlation with the firms asset value. The reason for this can be seen in Figure 5. Hedging in this case rapidly raises the value of the debt to its riskless level. This debt subsidy exceeds the value increase from avoiding bankruptcy, and the equity is made less valuable as a result of hedging.

Figures 7 - 10 and Figures 11 - 14 show that these effects hold under the alternative settlement rules, ANE and ANJ respectively. Thus the difficulty of generating viable hedging without costless bonding does not appear to be peculiar to the assumption of a particular settlement rule.

Figures 15 - 17 show the level of the forward rate spread, in basis points per annum. Figure 15 is the case where the hedging is reducing the risk of default, and gives a spread of a few basis points per annum for levels of hedging that minimise the probability of default. Figure 17, however, represents the case where the "hedging" is in fact speculation, and gives spreads that are higher by an order of magnitude. In the latter case, it can also be seen that the rule used to settle the claim is very important when the default risk is large.

## SECTION 7: Summary and Conclusions

We have developed a model of hedging when forward contracts are subject to default risk. We have shown how the forward contract rate and the values of debt and equity vary with the amount of hedging. We have identified the characteristics of firms the will find it profitable to hedge, and shown how the optimal size of the hedge relates to the minimum variance hedge.

We discover that the ability of debt holders to bond the hedging behaviour of the firm is crucial to the amount of hedging undertaken. Without bonding, some firms will not be able to use forward contracts to hedge in a way that satisfies both bondholders and

shareholders. In particular, low grade firms may suffer from this problem and be forced to use alternative hedging instruments, such as futures. The model we use is limited in that the benefit of hedging is to avoid a pure bankruptcy cost rather than some more general cost of financial distress. Most of the results obtain, however, with a more general specification of distress costs.

The paper has two important empirical implications. It predicts a relationship between bid/ask spreads in forward markets and bid/ask spreads in debt markets. This relationship depends upon settlement and priority rules and fundamental characteristics of hedging firms such as their asset volatilities. An important feature of the model, from an empirical point of view, is that it relates the forward spread to the net asset volatility of the hedging firm including the impact of the hedge on volatility. It is this volatility that is potentially observable through the share price of the firm, and not the gross volatility of the assets excluding the impact of hedging.

The second empirical implication of the paper is that the model predicts which firms will use the forward markets to hedge and which will not. In particular, high quality, low leverage firms with values closely correlated with the hedging instrument will use the forward markets. Other firms will, presumably, either not hedge or use some other instrument.

**Appendix :** Proof that there is no viable hedging strategy when the bank selling the forward contract holds no debt.

From proposition 2: for viable hedging:

$$P_r(V_T < D) > P_r(V_T + FX_T < D + FX_F) \quad (A1.1)$$

where:

$P_r(V_T < D)$  is the risk-adjusted probability that  $V_T < D$

Thus viable hedging must result in a decreased probability of bankruptcy.

This result is independent of the distributional assumptions about  $V_T$  and  $X_T$ . We now impose the normality assumption. A1.1 is equivalent, under this assumption, to:

$$N_0 > N_1 \quad (A1.2)$$

which implies:

$$(D - M_V) / S_V > (D - M_V + F(X_F - M_X)) / S_Z \quad (A1.3)$$

rearranging:

$$(M_V - D) / S_V < (M_V - D - (FX_F - FM_X)) / S_Z \quad (A1.4)$$

Using  $X_F > M_X$  gives:

$$M_V - D > M_V - D - (FX_F - FM_X) \quad (A1.5)$$

So:

$$S_Z < S_V \quad (A1.6)$$

Thus viable hedging requires that the total variability of  $V_T + FX_T$  is less than the total variability of  $V_T$ , so that  $X_T$  must be negatively correlated with  $V_T$ .

The condition that the equity value rises as a result of hedging is:

$$C(V_T + FX_T, D + FX_F) > C(V_T, D) \quad (A1.7)$$

Given (A1.6):

$$C(V_T + FX_T, D + FX_F) < C(V_T, D + FX_F - FM_X) \quad (A1.8)$$

Given  $FX_F > FM_X$ :

$$C(V_T, D + FX_F - FM_X) < C(V_T, D) \quad (A1.9)$$

Which contradicts A1.7, so viable hedging is not possible at a positive spread.

This proves P7. P6 follows from the fact that the spread will be positive if the bank granting the forward contract holds no debt.

1. We restrict our attention to hedging currency risk using forward contracts so that we can use no-arbitrage pricing to evaluate the contingent claims arising in the hedging. Those results of the paper that do not rely upon continuous trading of the instrument underlying the forward contract would also, presumably, apply to hedging other exposures using forward contracts. All the results of the paper could also be extended to apply to hedging interest rate risk using FRA's and swaps if the model were extended to include stochastic interest rates.
2. Although we analyse a pure bankruptcy cost, where the deadweight loss is incurred only if the firm goes bankrupt, the first nine positions, P1 to P9, hold true also if there are more general financial distress costs. Thus if the firm suffers a fixed deadweight loss,  $K$ , when its assets value falls below some multiple of the fixed claims on the firm, propositions P1 to P9 still obtain.
3. If the forward contract is senior to the debt, or matures prior to the debt with no debt covenant to protect the bondholders' claim from dilution, the spread on the forward contract will be reduced, and the other results of the paper will be affected. We leave this case for later analysis.
4. The fact that the hedge must reduce the risk-adjusted probability of bankruptcy does not mean that it must reduce the actual probability of bankruptcy. Whether it does will depend upon the prices of risk of the asset,  $V$ , and the currency,  $X$ .
5. In this case, we are measuring the forward rate spread from the difference between the offer side of the forward market and the mid-market interest rate parity rate. If we wished to get the bid/offer spread we would analyse the transaction where the firm pays  $FX_T$  and receives  $FX_F$  in the forward market, solve for the equilibrium rate and this would give us the bid quote.

Table 1: No Hedging. Payoffs at Time T to Debt and Equity

State	Debt	Equity
$V_T < D$	$V_T - K$	0
$V_T > D$	D	$V_T - D$



Table 2: Event Sequence and Payoffs: Gross Settlement (Assumption AG)

2A:	Event Sequence		
	Time	0	T
	Event	Forward contract initiated	a) Bank pays $FX_T$ b) If $V_T + FX_T < D + FX_F$ assets shrink by $K$ c) Claims settled

2B: Payoffs at time T

State	Debt payoff
1	$[D/(D+FX_F)] (V_T+FX_T-K)$
2	$D$
State	Forward payoff
1	$-FX_T + [FX_F/(D+FX_F)] (V_T+FX_T-K)$
2	$-FX_T + FX_F$
State	Equity payoff
1	$0$
2	$V_T+FX_T-D-FX_F$
State	Total payoff
1	$V_T-K$
2	$V_T$

State definitions:

State 1:	$V_T+FX_T < D+FX_F$
State 2:	$V_T+FX_T > D+FX_F$

Table 3: Payoffs: Net settlement, Equal Priority (Assumption ANE)

State	Debt Payoff
1	$D$
2	$V_T + FX_T - FX_F - K$
3	$D$
4	$[D / (D + FX_F - FX_T)](V_T - K)$
State	Forward Payoff
1	$-FX_T + FX_F$
2	$-FX_T + FX_F$
3	$-FX_T + FX_F$
4	$[(FX_F - FX_T) / (D + FX_F - FX_T)](V_T - K)$
State	Equity Payoff
1	$V_T + FX_T - D - FX_F$
2	$O$
3	$V_T + FX_T - D - FX_F$
4	$O$
State Definitions:	
State 1:	$FX_T > FX_F$ and $V_T + FX_T > D + FX_F$
State 2:	$FX_T > FX_F$ and $V_T + FX_T < D + FX_F$
State 3:	$FX_T < FX_F$ and $V_T + FX_T > D + FX_F$
State 4:	$FX_T < FX_F$ and $V_T + FX_T < D + FX_F$

Table 4: Payoffs: Net Settlement, Forward Contract Junior (Assumption ANJ)

State	Debt Payoff
1	$D$
2	$V_T + FX_T - FX_F - K$
3	$D$
4	$D$
5	$V_T - K$

State	Forward Payoff
1	$-FX_T + FX_F$
2	$-FX_T + FX_F$
3	$-FX_T + FX_F$
4	$V_T - D - K$
5	$O$

State	Equity Payoff
1	$V_T + FX_T - D - FX_F$
2	$O$
3	$V_T + FX_T - D - FX_F$
4	$O$
5	$O$

State Definitions:

State 1:	$FX_T > FX_F$ and $V_T + FX_T > D + FX_F$
State 2:	$FX_T > FX_F$ and $V_T + FX_T < D + FX_F$
State 3:	$FX_T < FX_F$ and $V_T + FX_T > D + FX_F$
State 4:	$FX_T < FX_F$ and $V_T + FX_T < D + FX_F$ and $V_T > D + K$
State 5:	$FX_T < FX_F$ and $V_T + FX_T < D + FX_F$ and $V_T < D + K$

## References:

- Cox, John C and Stephen A Ross, 1976, The Valuation of Options for Alternative Stochastic Processes, *Journal of Financial Economics* 3, 145-166.
- Cox, John C, Stephen A Ross and Mark Rubinstein, 1979, Option Pricing: A Simplified Approach, *Journal of Financial Economics* 7, 229-263
- Levi, Maurice and J. Zechner, 1989, Foreign Exchange Risk and Exposure, in Robert Z. Aliber (ed) *The Handbook of International Financial Management*, Dow Jones Irwin, 1989
- Merton, Robert C, 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance* 29, 449-470.
- Press, William H, Brian P Flannery, Saul A. Tenkolsky and William T. Vetterling, 1986, *Numerical Recipes*, Cambridge University Press
- Smith Clifford W and Rene M Stulz "The Determinants of Firms' Hedging Policies" *Journal of Financial and Quantitative Analysis*, Vol 20 No 4, December 1985, 391-405

Figure 1 : Assumption AG, Normal Distribution

TOTAL VALUE: ASSETS=100, DEBT=50,

MAI=5, SIGV=40%, SIGA=15%, R=10%

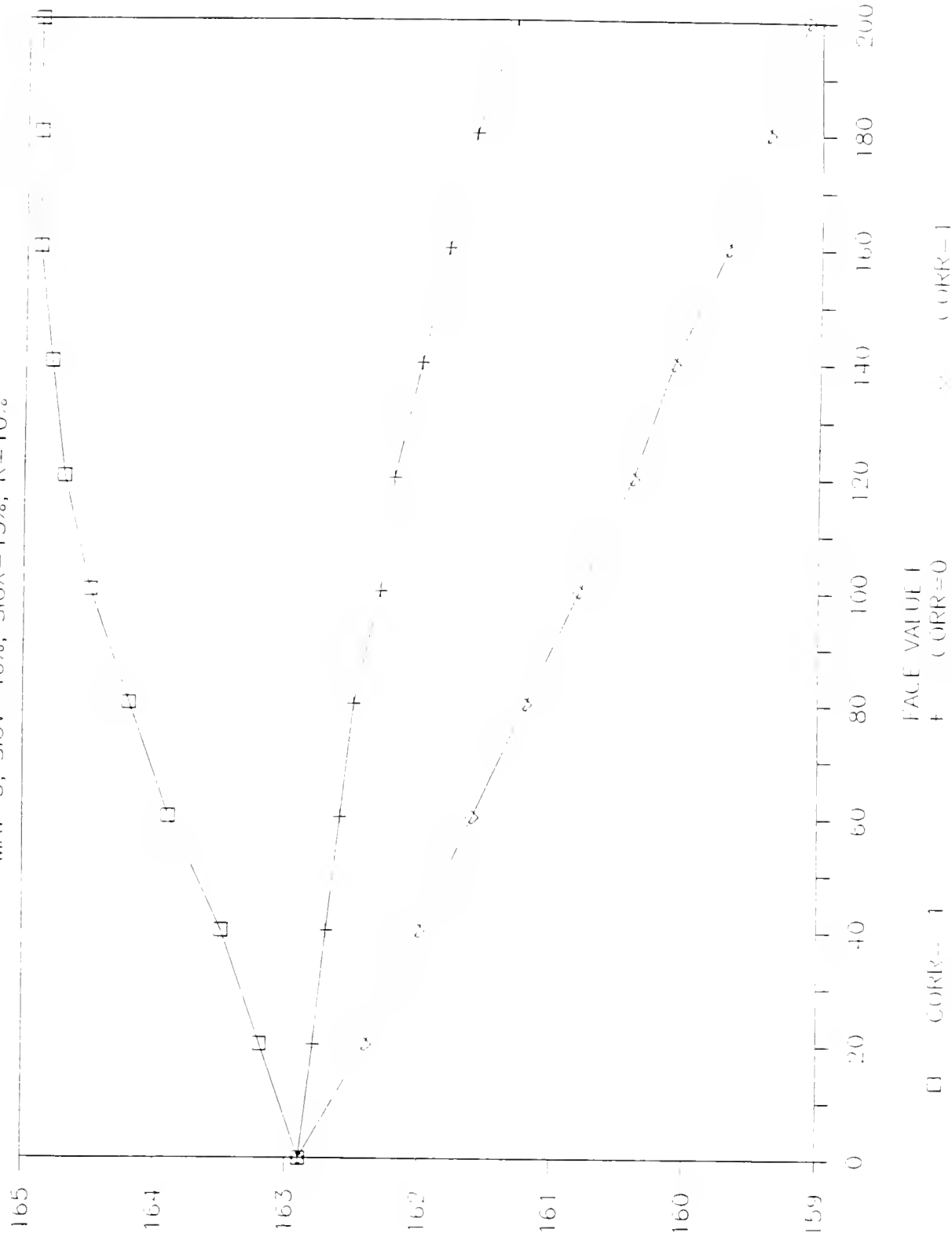


Figure 2 : Assumption AG, Normal Distribution

LOG10 VALUE: ASSET= 100, DFB1=50,

MA=5, SIGV=40%, SIG=15%, K=10%

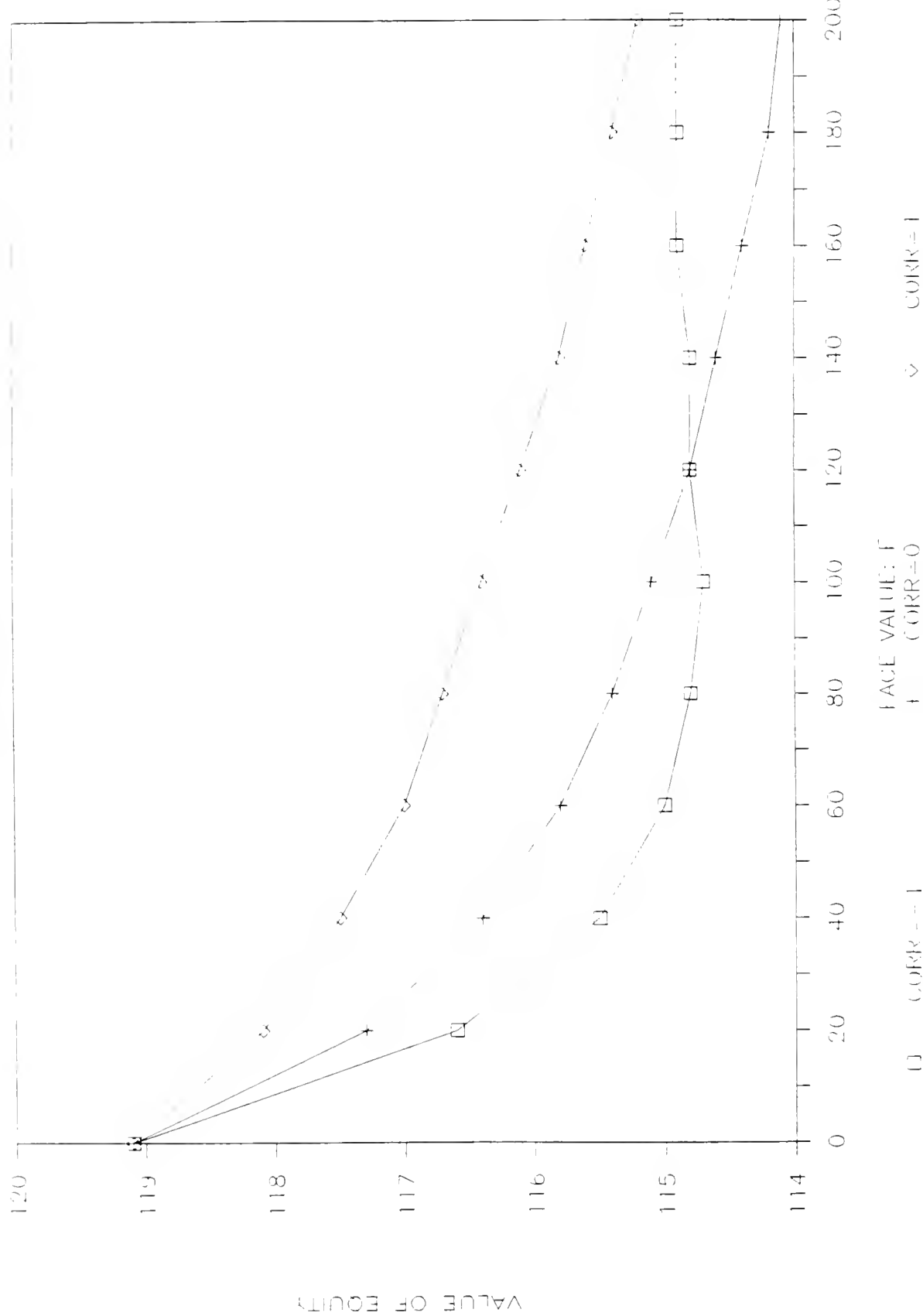


Figure 3 : Lognormal Distribution

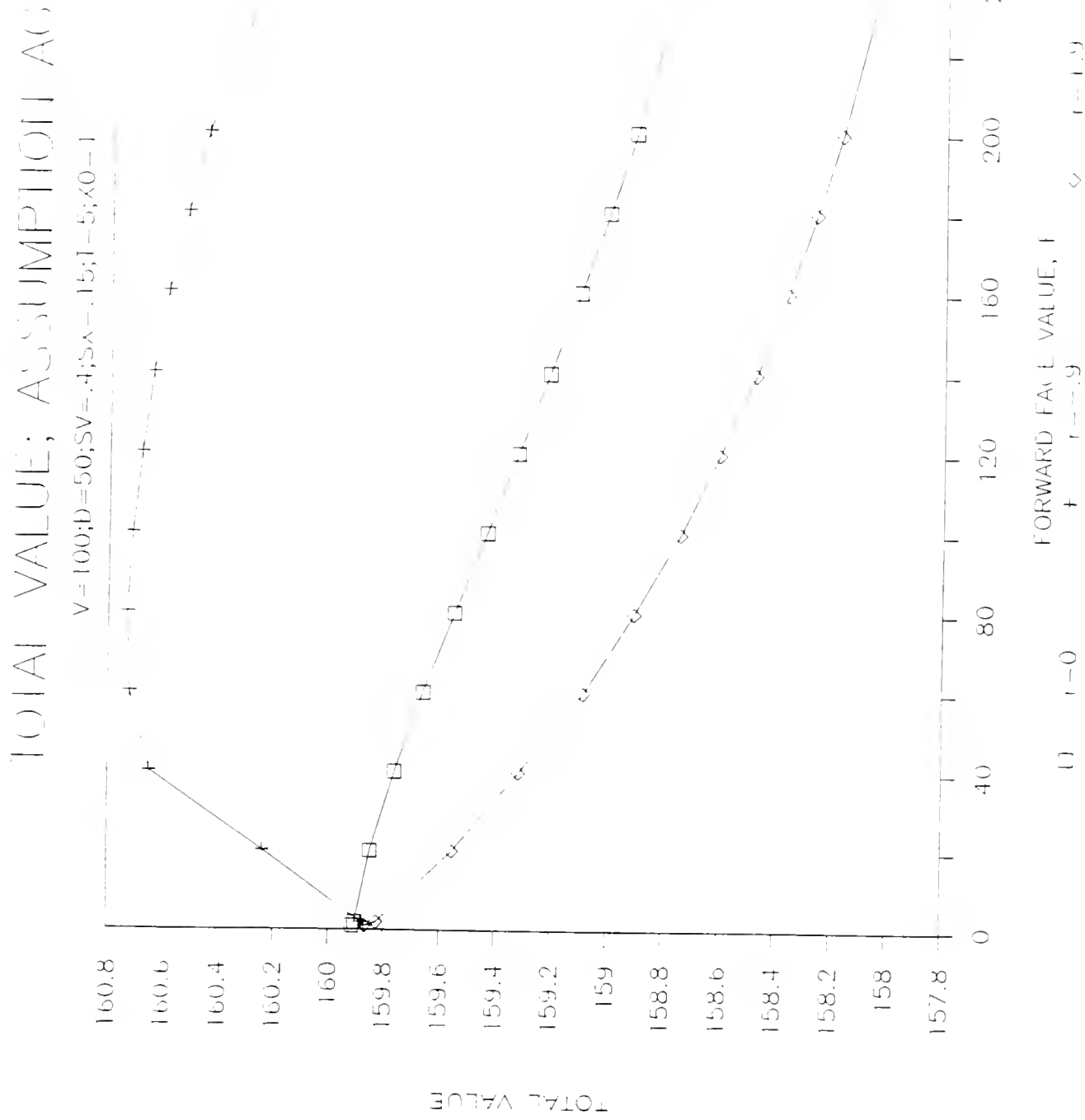


Figure 4 : Lognormal Distribution

# EQUITY VALUE; ASSUMPTION A,

$V=100; D=50; SV=.4; S_r = .15; T = 5; X_0 = 1$

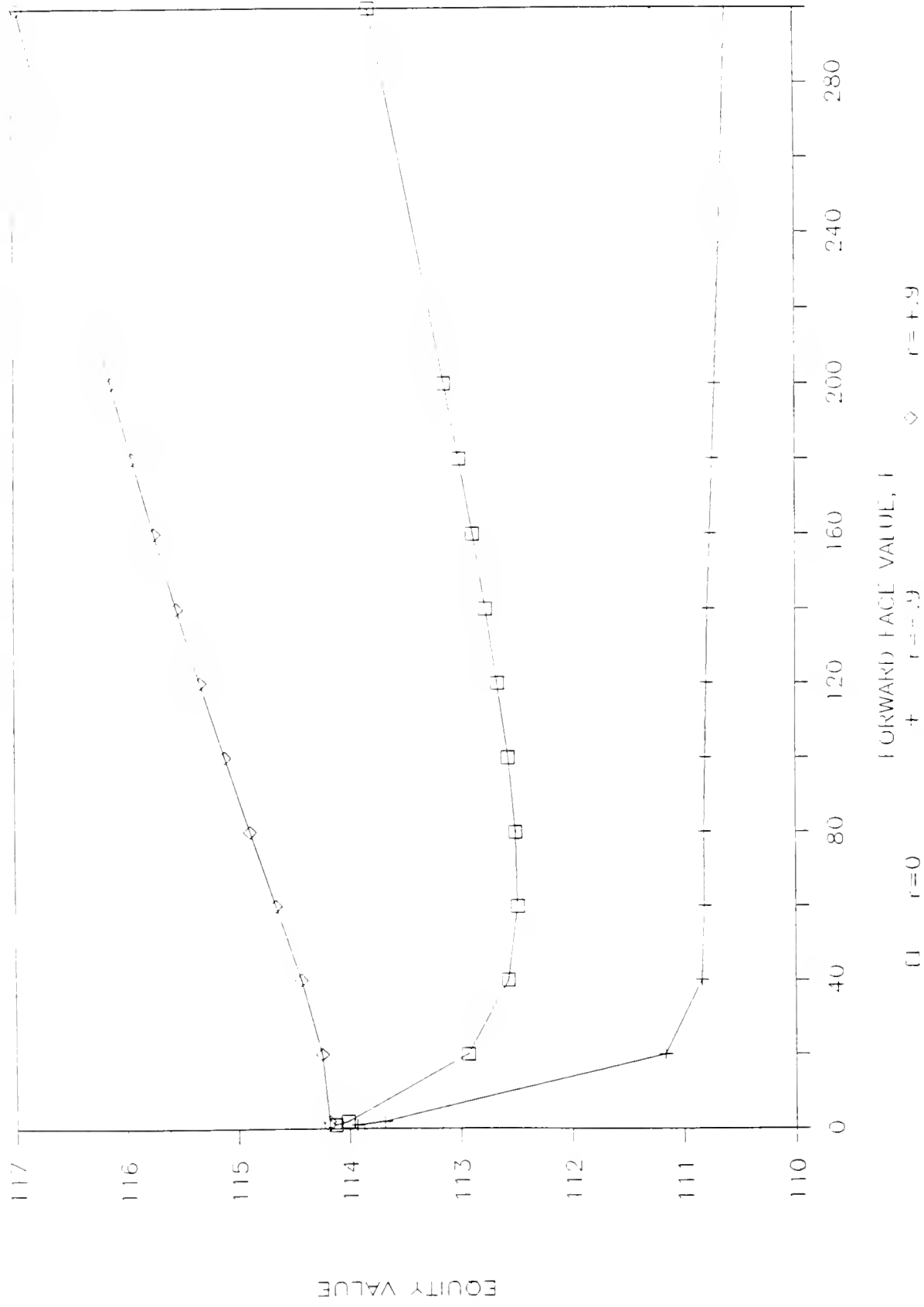




Figure 5 : Lognormal Distribution

# DEBT VALUE; ASSUMPTION AG

$V = 100; D = 50; SV = .4; \delta = .15; T = 5; \chi_0 = 1$

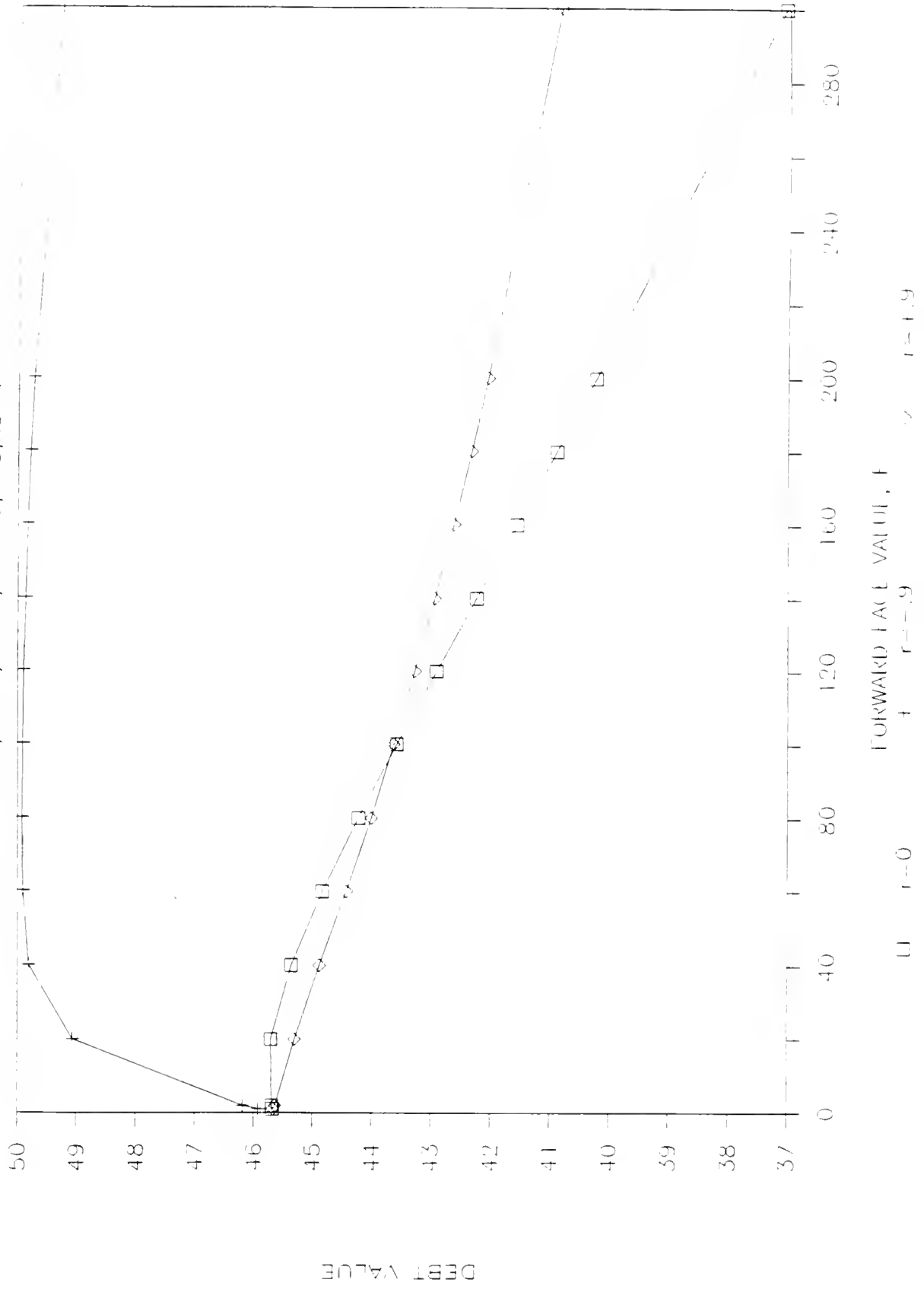


Figure 6 : Lognormal Distribution

CONTRACT RATE,  $r$ ; ASSUMPTION  $A_0$

$V=100; D=50; S_V=.4; S_r=.15; 1-5; \alpha_0=1$

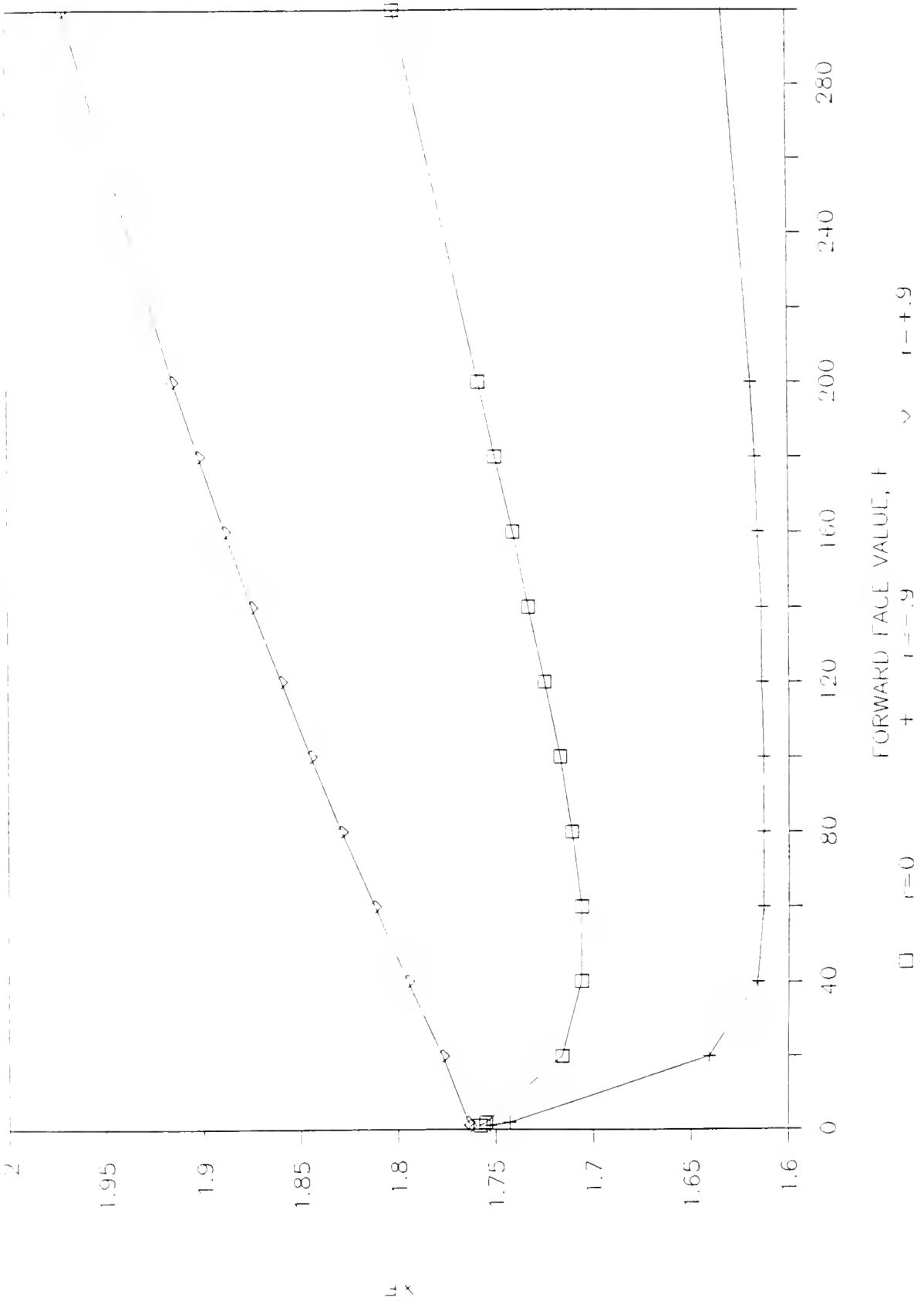


Figure 7 : Lognormal Distribution

# TOTAL VALUE; ASSUMPTION ANH

$V=100; D=50; SV=.4; SX=.15; T=5; X_0=1$

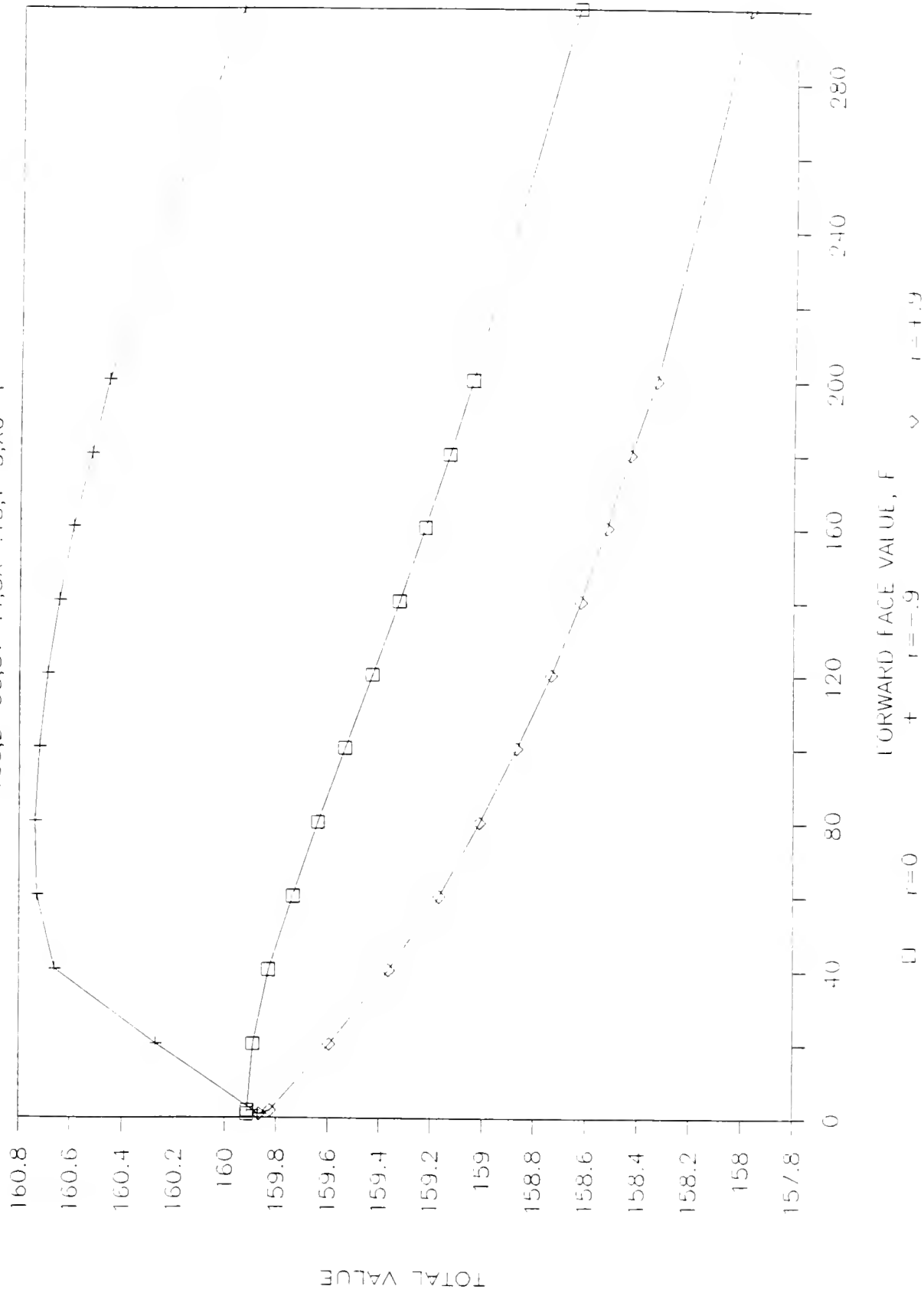


Figure 8 : Lognormal Distribution

# EQUITY VALUE; ASSUMPTION AIII

$V=100; D=50; SV=.4; \sigma_A=.15; T=5; \lambda_0=1$

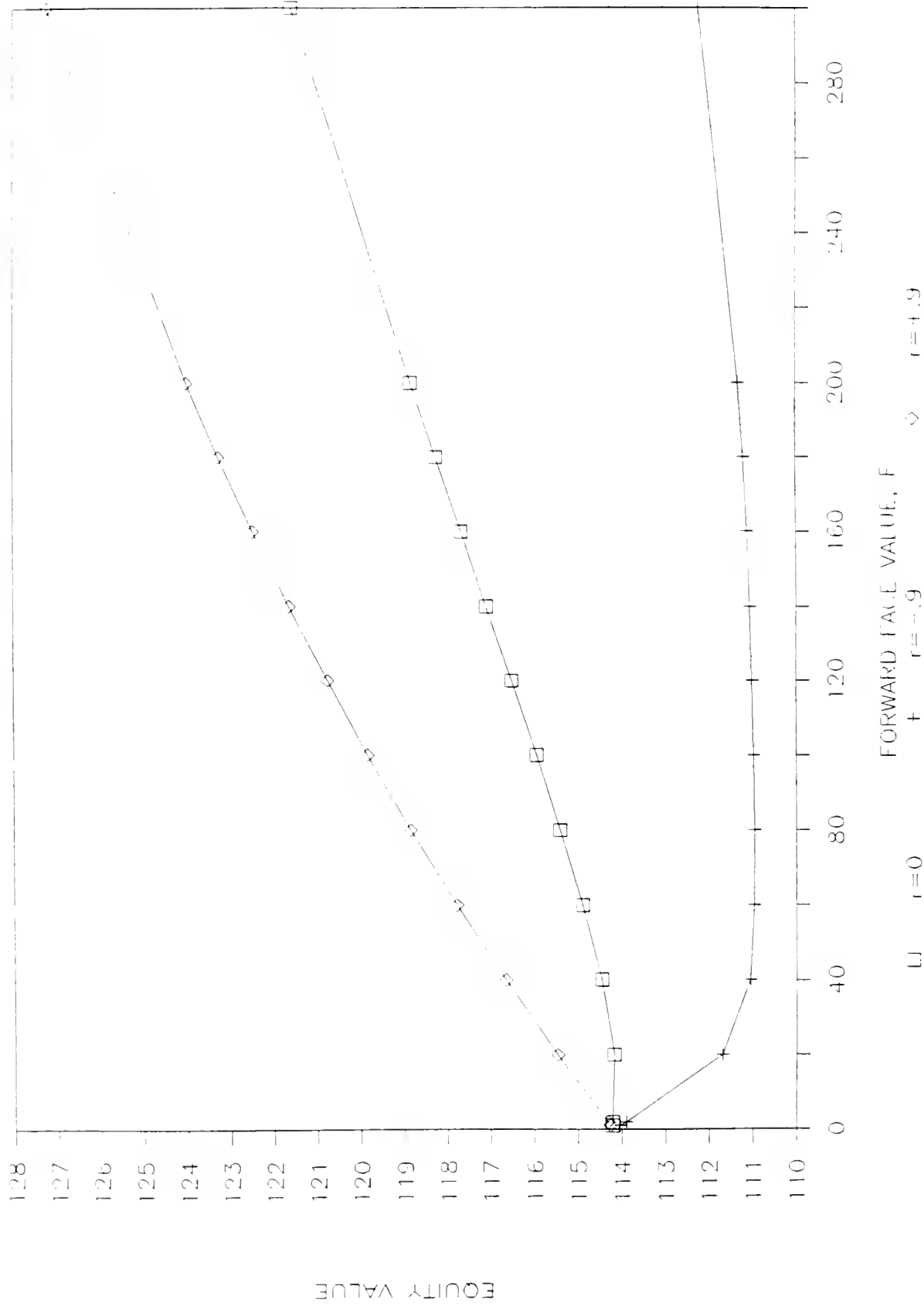


Figure 9 : Lognormal Distribution

# 10131 VALUE; ASSUMPTION A11

V 100;D: 50;M - 4;S - .15;1 - 5;0 = 1

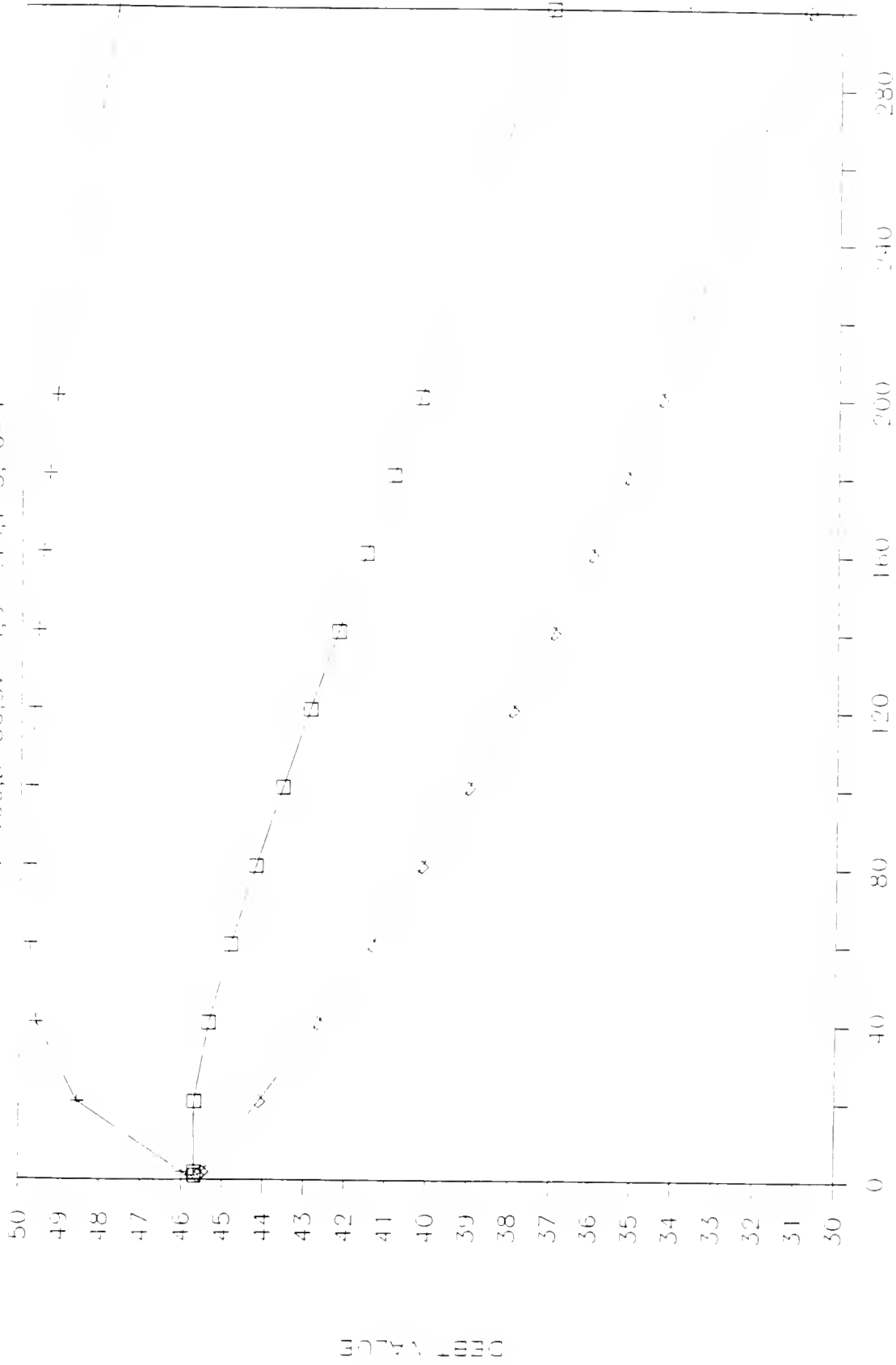


Figure 10 : Lognormal Distribution

CONTRACT RATE,  $AF$ ; ASSUMPTION ALL

$V = 100; D = 50; SV = 4; S^2 = 15; I = 5; r = 1$

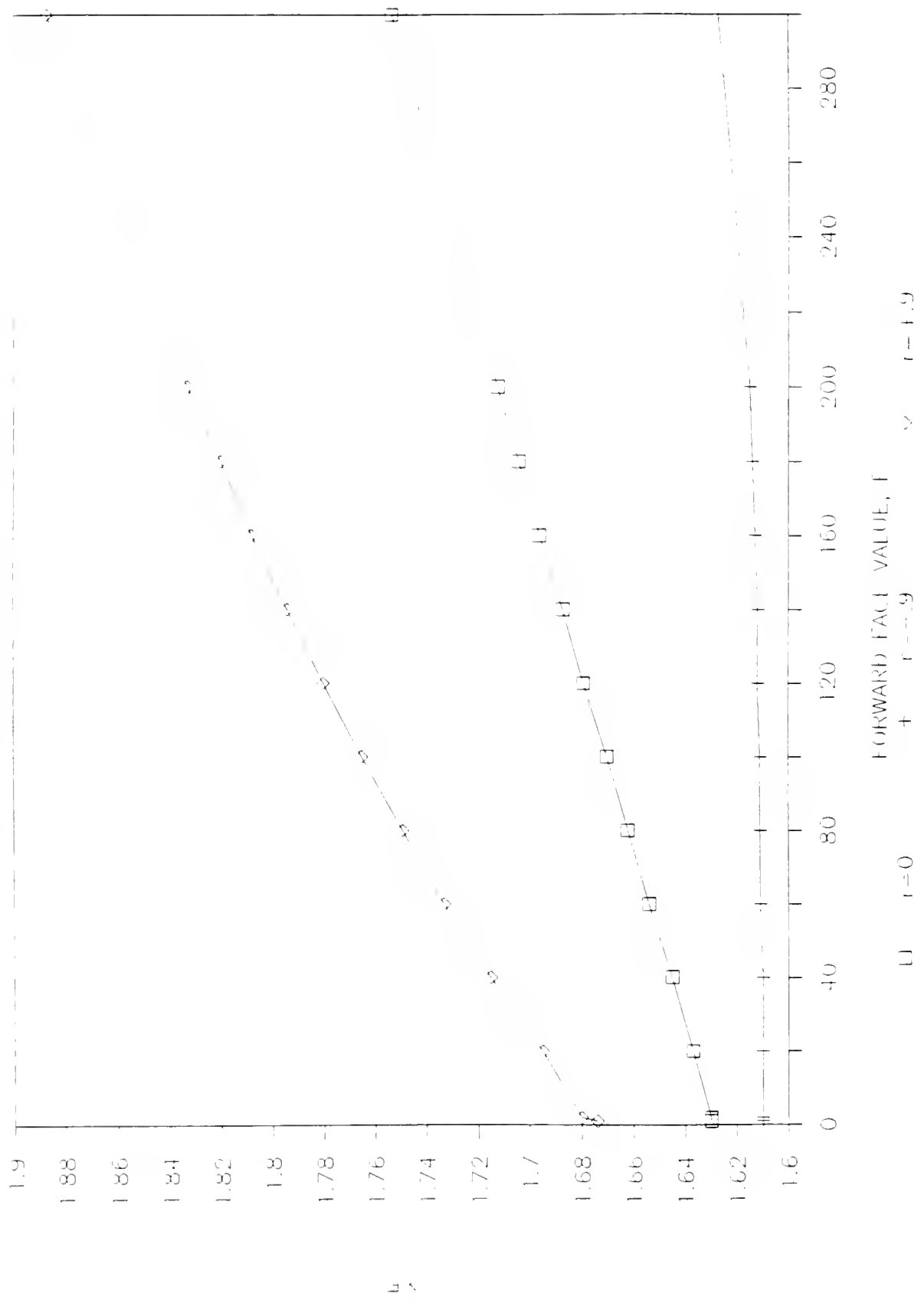


Figure 11 : Lognormal Distribution

# TOTAL VALUE; ASSUMPTION A11J

V=100;D=50;SV=.4;S=.15;T=5;X0=1

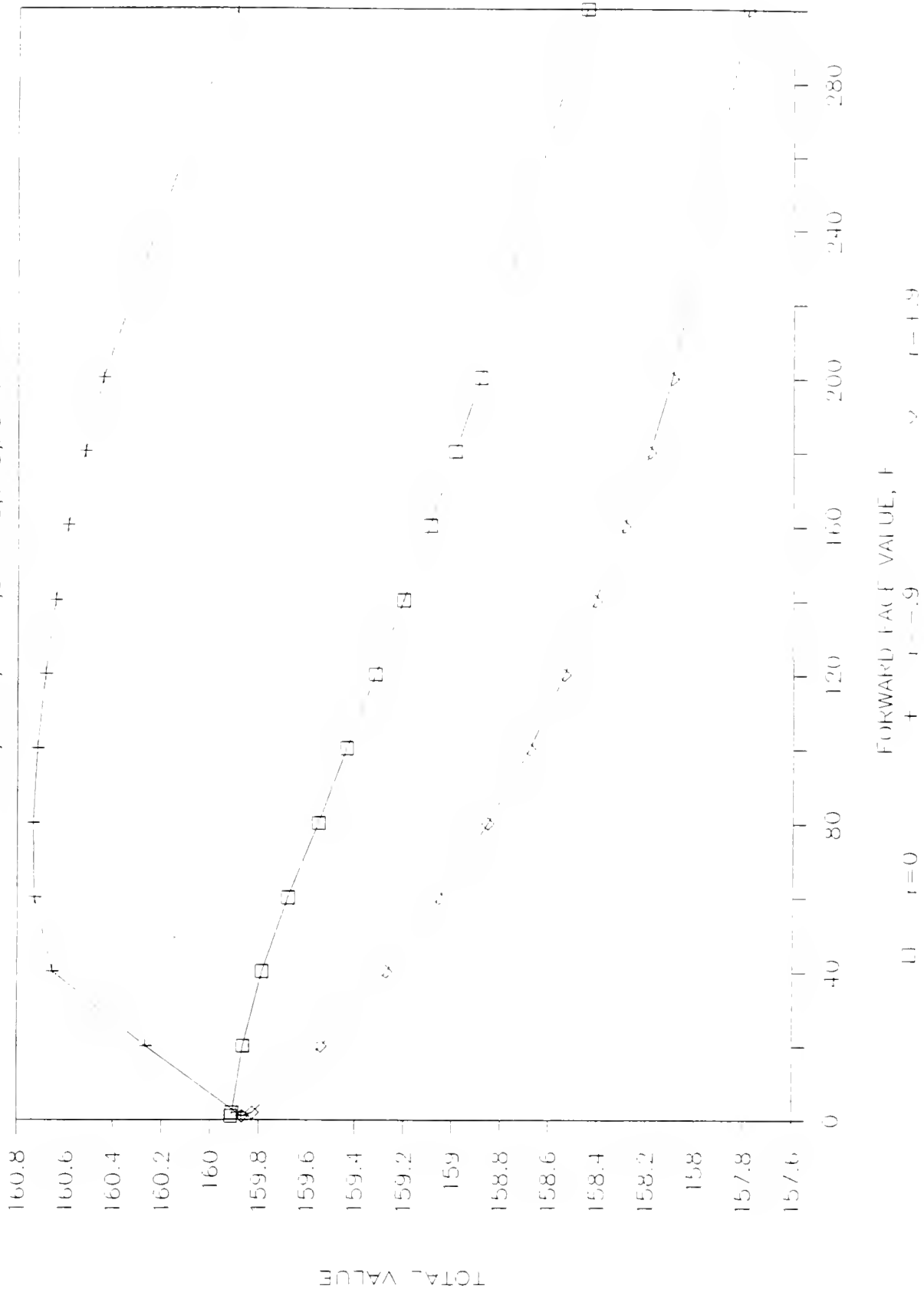


Figure 12 : Lognormal Distribution

## EQUITY VALUE; ASSUMPTION A111

$V=100; D=50; SV=4; SA=15; T=5; r=1$

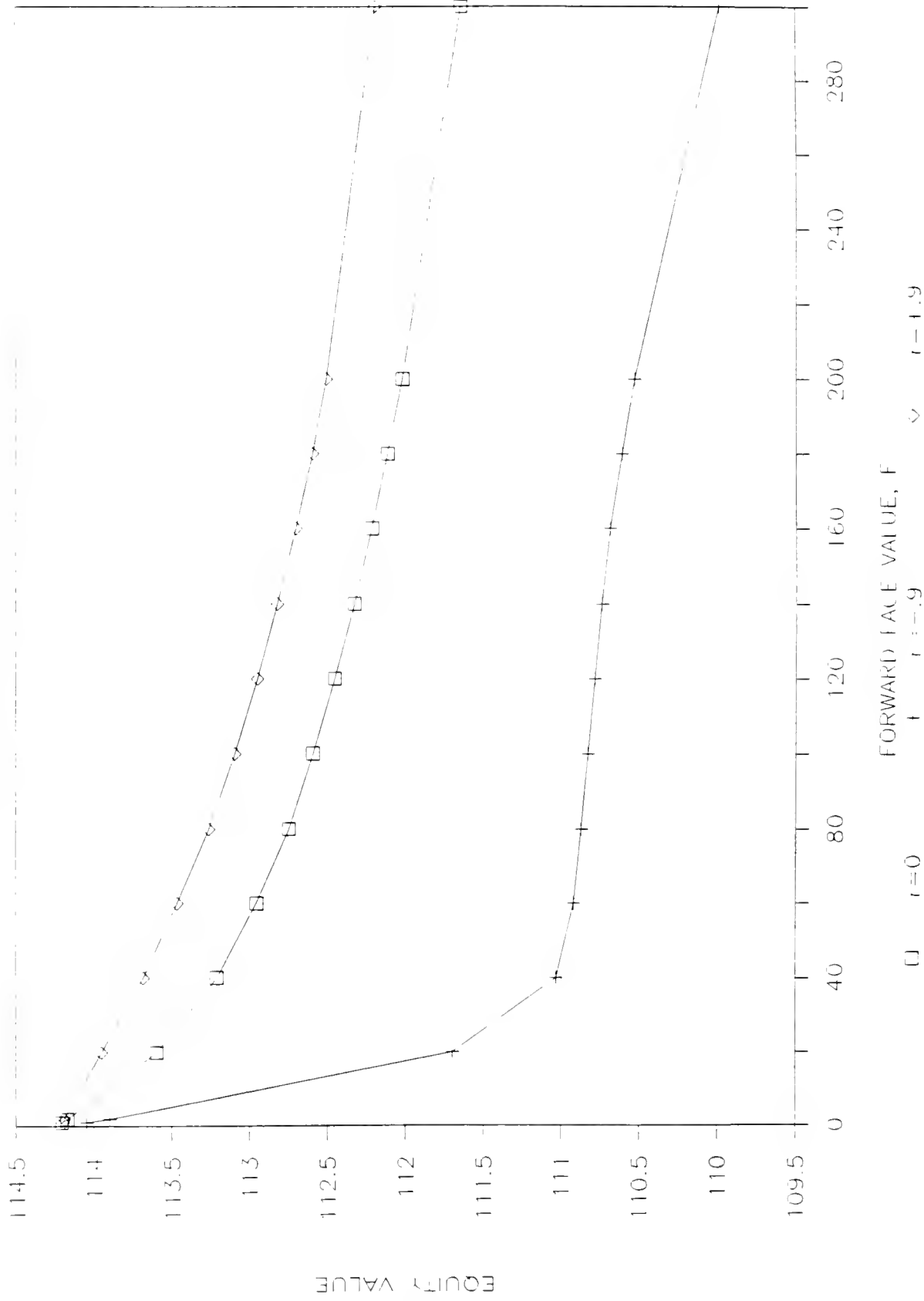




Figure 13 : Lognormal Distribution

# DEBT VALUE; ASSUMPTION A11.1

$V = 100; D = 50; SV = .4; S_A = .15; I = 5; \lambda = 0 - 1$

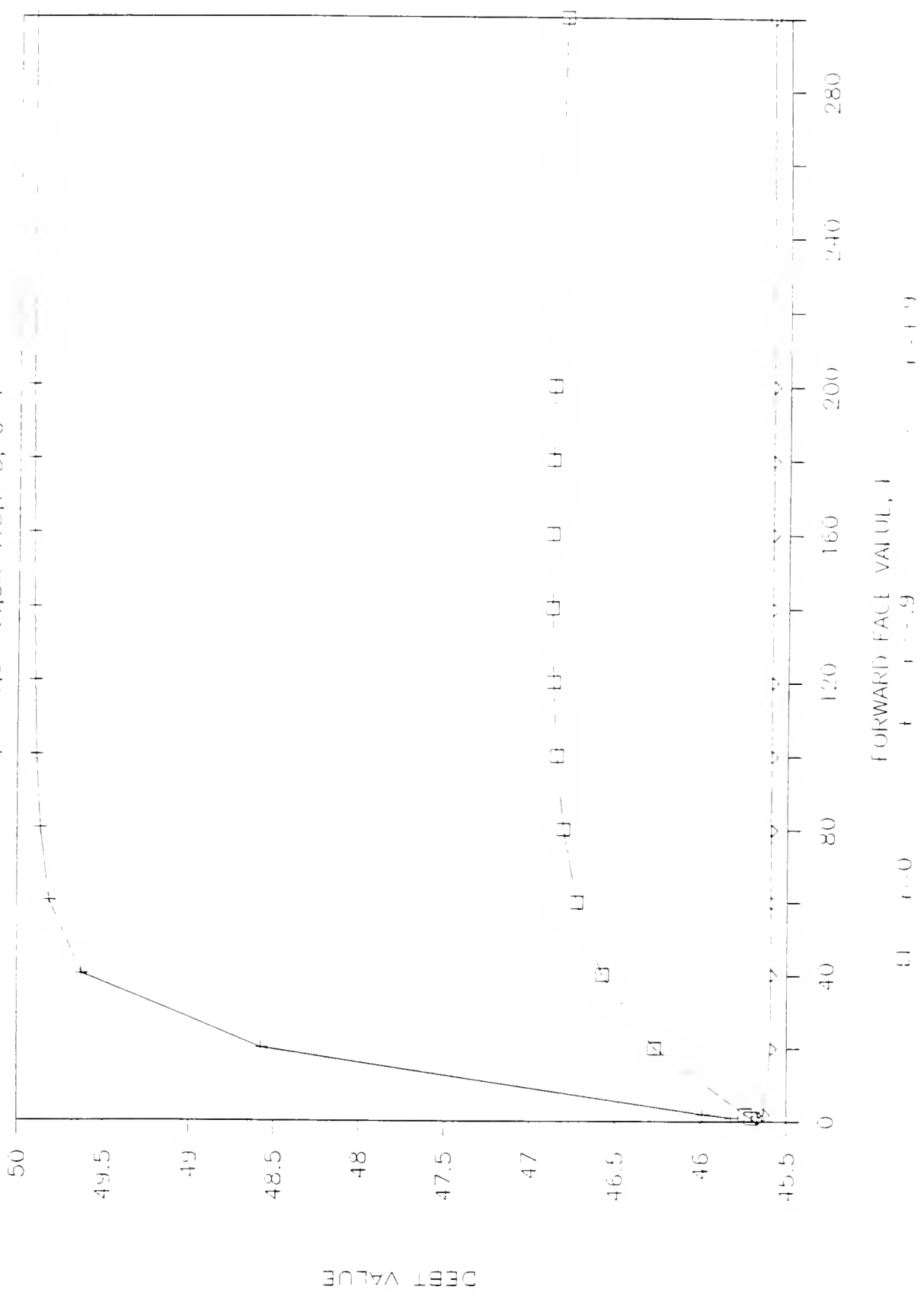


Figure 14 : Lognormal Distribution

CONTRACT RATE, XF; ASSUMPTION ALL

$V = 100; D = 50; SV = .4; S^2 = .15; T = 5; \lambda_0 = 1$

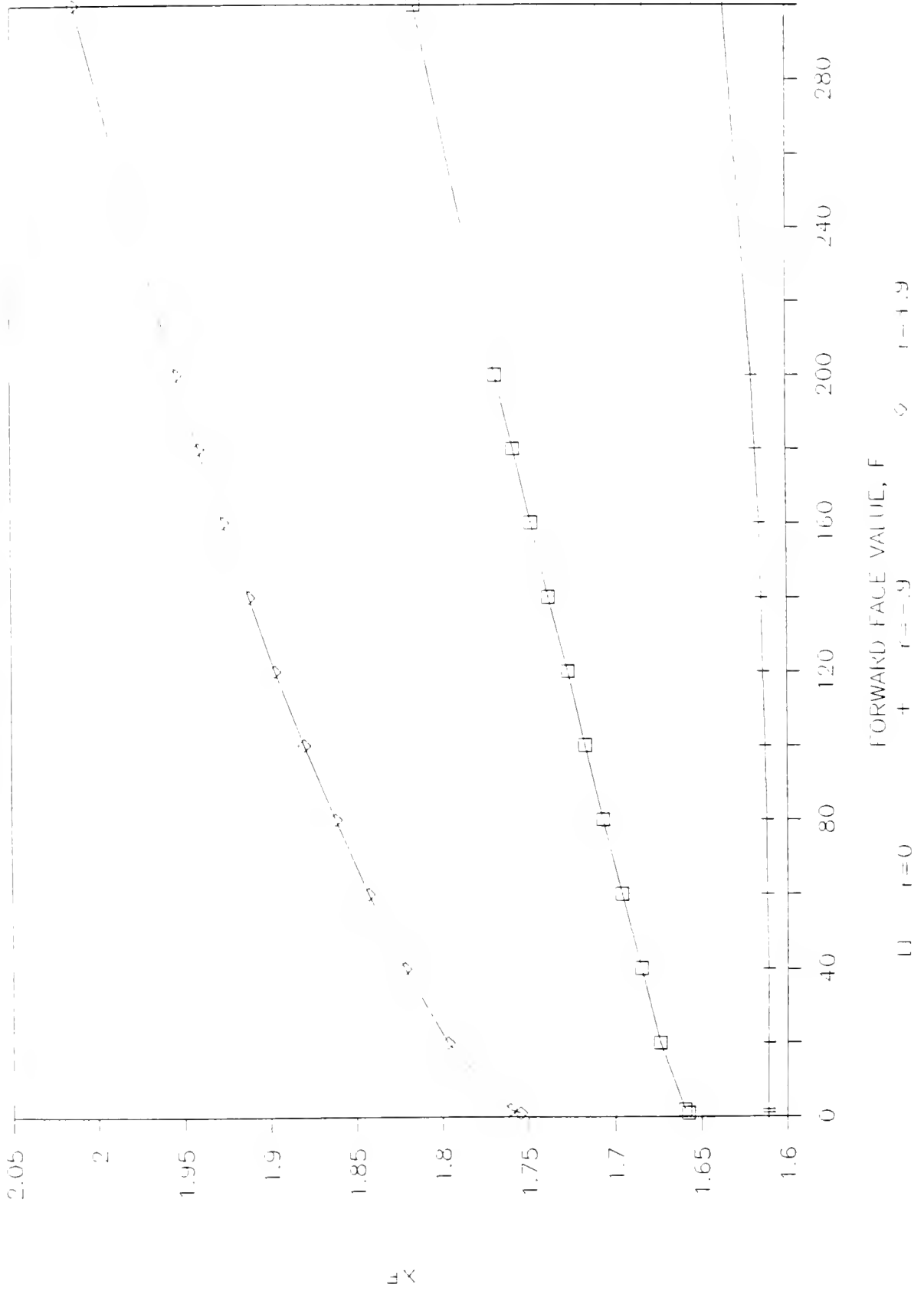


Figure 15 : Lognormal Distribution

# FORWARD RATE SPREAD

CORR = .9

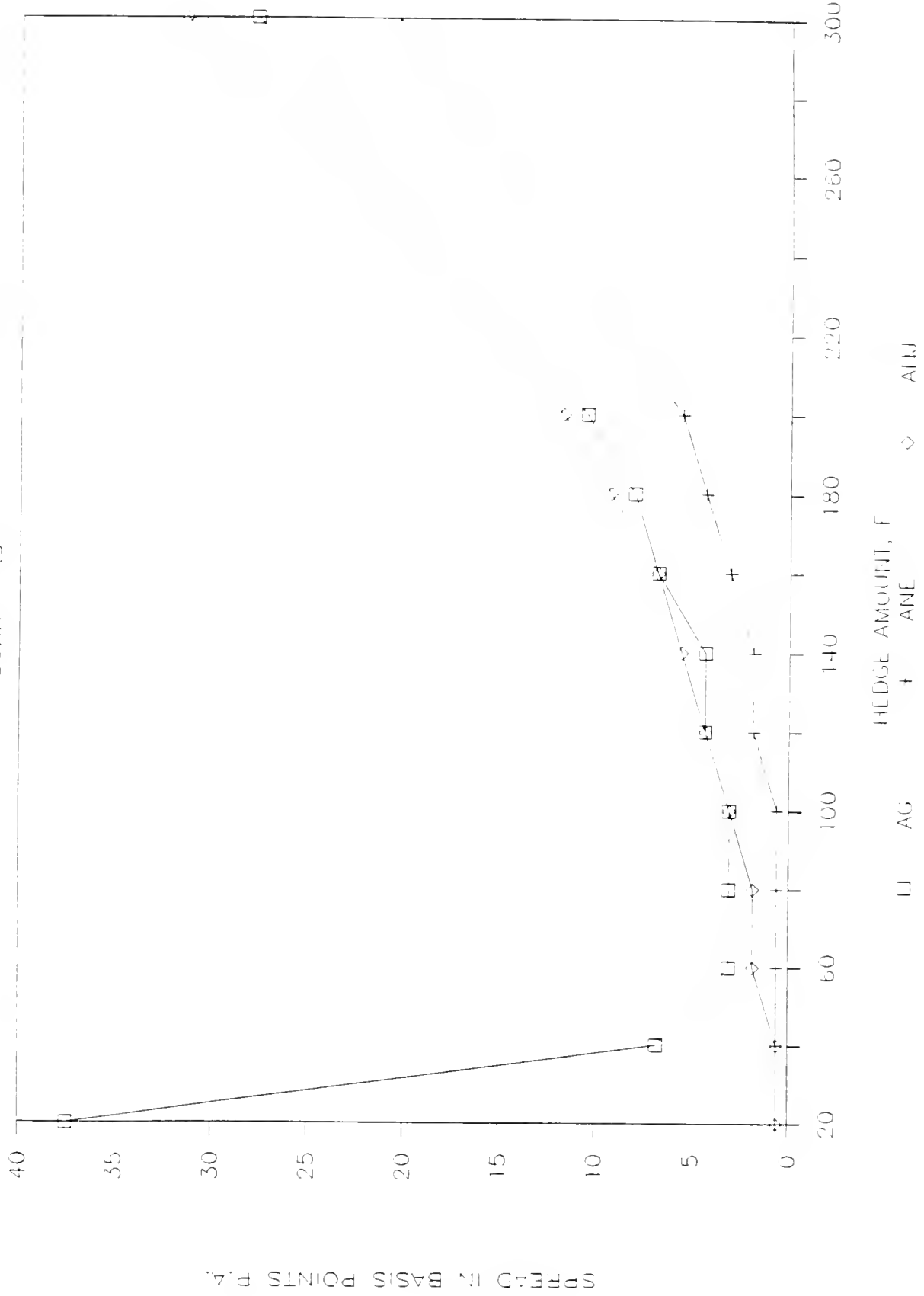


Figure 16 : Lognormal Distribution

# FORWARD RATE SPREAD

CORR= 0

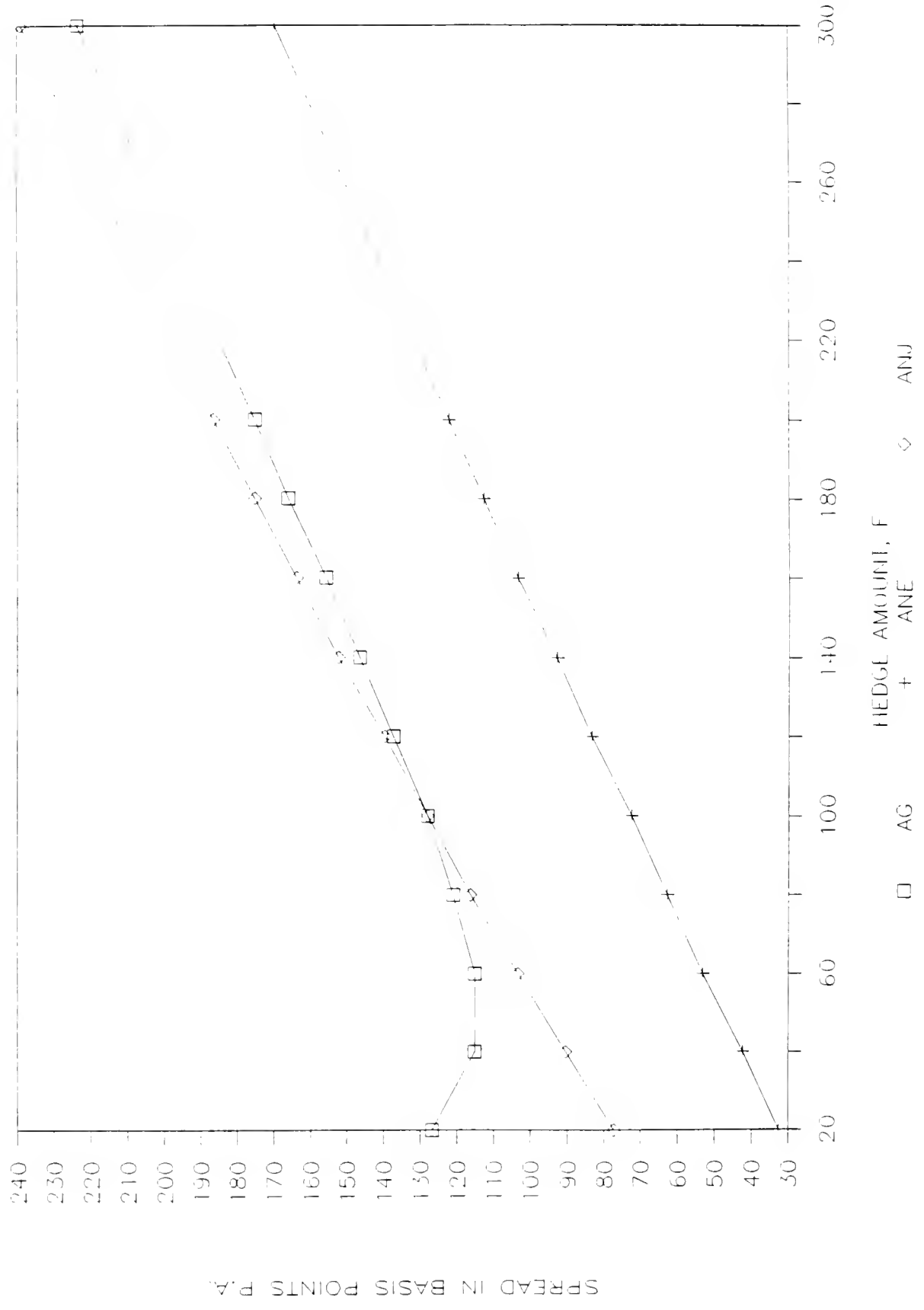
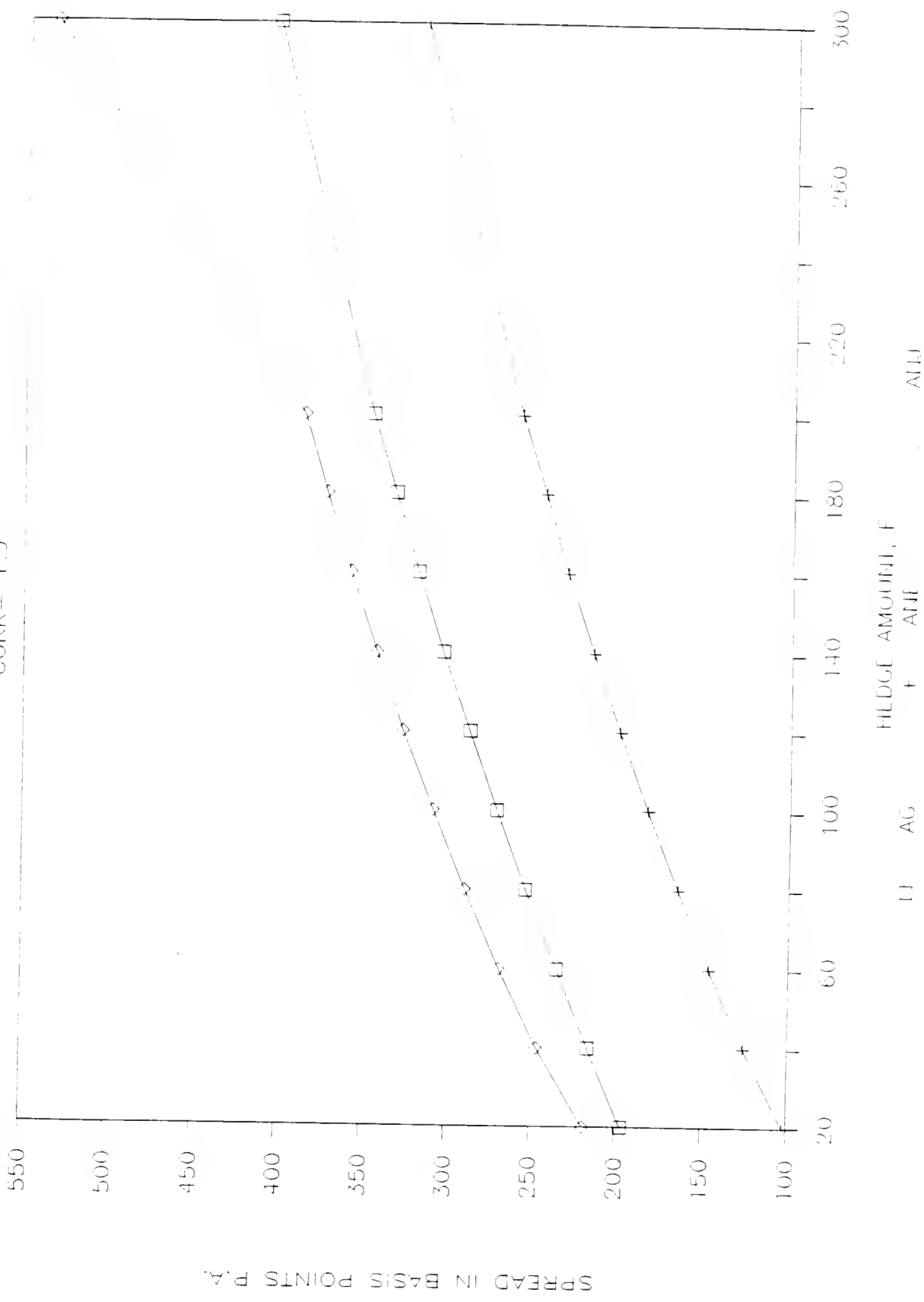


Figure 17 : Lognormal Distribution

## FORWARD RAIL SPREAD

CORR = 1.0







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